Graphs, trees and spanning trees

* Graphs consist of vertices and edges, representing relationships. They can be visualized as dots connected by lines.
* If each dot in the graph is represented by letters, then it is a labelled graph other wise it is an unlabeled graph.
* There are different types of characteristics for a graph, such as; subgraphs, induced subgraphs, vertex degrees and handshake lemma.
* There are different types of named graphs; Kn, Km,n, Cn, Pn. Each specifies a different type of graph.
* A tree is a connected graph containing no cycles. A forest is a graph containing no cycles. Note that this means that a connected forest is a tree.
* Properties of trees:
  + Trees have a unique path between any pair of distinct vertices.
  + Forests have at most one path between any pair of vertices.
  + Trees with v vertices and e edges satisfy e = v − 1.
* Rootes trees are trees with specified root vertex. There is a hierarchy in rooted trees (grandparent, parent, child).
* Spanning trees are sub graphs of connected graphs.
* This concept is mostly useful in path-finding algorithms, data structures, in social and computer networking, problem solving, mathematics and many more.

Planer graphs

* Planar graphs are those that can be drawn on a plane without edge crossings whereas the non-planar are the vice versa.
* Did proof by contradiction in planar representation using Euler’s formula.
* Then there was introduction to polyhedral, which explored the 3D objects, polyhedral, and their connection to planar graphs.
* This concept is widely used in Map design and circuit layouts, Network planning, geometry and architecture, algorithm design, and much more.

Euler trails and Euler circuits

* Euler trails and circuits defined in graphs and multigraphs.
* Euler trail: Walk through a graph using every edge exactly once.
* A graph has a Euler circuit if and only if there are zero or two vertices with odd degree.
* Euler circuit: Euler trail that starts and stops at the same vertex.
* A graph has a Euler circuit if and only if the degree of every vertex is even.
* Fleury's algorithm helps find Euler trails or circuits in Eulerian graphs.
* Input: Eulerian graph G, choose odd-degree starting vertex, follow edges, record, and erase edges.
* It is used mainly used in Network design, circuit planning, algorithmic problem solving, graph theory application.

References

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